Maximum T(q)-Likelihood Estimation: a New Method and its Application in Risk Management

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Risk Management is a procedure for shaping a loss distribution. Let $X_1, ..., X_n$ be a iid sample from a loss distribution with a density $f(x, \theta_0)$, where θ_0 is unknown (vector) parameter. The Maximum T(q)-Likelihood Estimator (MT(q)LE) $\tilde{\theta_n}$ of θ_0, is defined as

$$\tilde{\theta_n} = argmax_{\theta \in \Theta} \sum_{i=1}^{n} T(q, f(X_i; \theta)), \quad q > 0,$$

where

$$T(q, u) = \begin{cases} \frac{u^{1-q}-1}{1-q}, & \text{if} \quad q \neq 1; \\ \ln u, & \text{if} \quad q = 1. \end{cases}$$

If $q \to 1$, then $T(q, u) \to \ln u$ and the usual MLE is recovered, i.e. the MT(q)LE extends the classic MLE method. For a fixed $q > 0$, the function $T(q, u)$ represents the Box-Cox transformation in statistics, often called deformed logarithm of order q. The MT(q)LE $\tilde{\theta_n}$ can be computed by $T(q)$-likelihood equation(s)

$$\frac{\partial}{\partial \theta} \sum_{i=1}^{n} T(q, f(X_i; \theta)) = 0.$$

Basic properties of the MT(q)LE will be shown. We will discuss the choice of the parameter q when evaluating VaR and CVaR for a real data set and by Monte Carlo simulations.